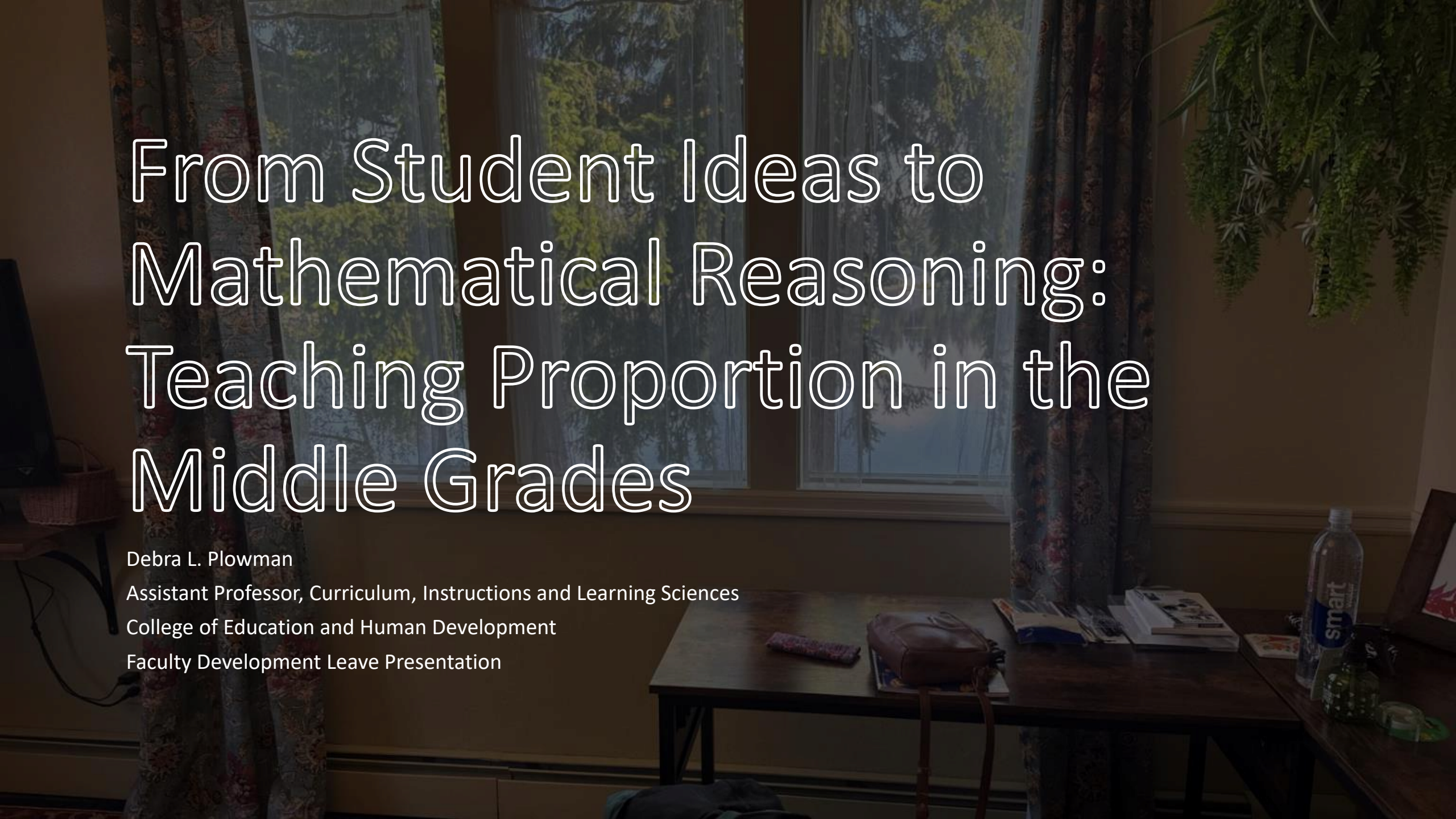

# From Student Ideas to Mathematical Reasoning: Teaching Proportion in the Middle Grades

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Faculty Development Leave Presentation

# Proportional Reasoning:

Jack buys 6 pencils for 6 dollars. Jill buys 12 pencils for 4 dollars. Who got the better deal?

A student who engages in  proportional reasoning will recognize that paying 2 dollars for 6 pencils is the same as paying 4 dollars for 12 pencils, as the cost doubles with the quantity. Or they might recognize that the number of pencils is three times the number of dollars or that 1 pencil is 3 dollars.

This reasoning is critical in middle grades (5th–9th), but typical instruction often fails to fully develop it, primarily because teachers lack a deep understanding of how to nurture students' reasoning about proportions.

Proportional reasoning in mathematics provides the foundation for understanding more advanced concepts in algebra, geometry, and calculus.

# Mathematical Understanding of Proportion for Teaching

Utilizing previous research on student strategies for these problems, my own studies in teacher learning and decision making and extensive work in professional development, the primary goal of my Faculty Development leave centered on putting together a book for teachers and teacher educators that would support them to teach proportion and proportional reasoning.

Teachers and their students  benefit from understanding--

- the meaning of proportion,
- the range of reasoning that can be used to solve problems involving proportion,
- and how and what to present to students during instruction to engage students to reason proportionally.

# The Original Plan

- Drawing on my research in teacher development, I planned to draft a book manuscript designed to help teachers effectively teach proportional reasoning. The book will provide problem examples that stimulate student thinking, along with strategies for lesson facilitation and class discussions to deepen understanding. ~~It will include student interviews and video-recorded lessons,~~ illustrating how to engage students in proportional reasoning.
- **Instead**-Collected more data from two schools to investigate strategies for the comparison problem type which has not been featured extensively in modern research or treated any differently than the missing value types.

# Examples: Missing Value and Comparison Problem Types

Research to Practice: Students strategies for two types of proportion problems, coupled with knowledge of the construction of those problems come directly from the research study. Teachers study and learn from student strategies and word problems to inform their own teaching.

The book content includes classroom vignettes, teacher-student interaction examples, planning tasks, problem sets and exercises, and the research background.

Example 1:

A Missing Value Proportion Problem asks the problem solver to find the missing value of one quantity when the three other quantities are known.

A group of aliens* are planning a trip to Earth.

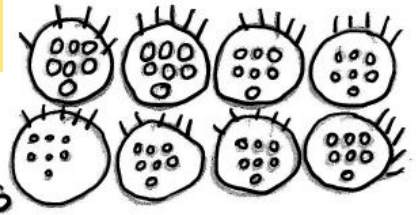
For every 5 aliens they need 7 Power Pouches in order to meet one day´s food needs.

If there are 40 aliens traveling, how many Power Pouches will they need for each day of the trip?

*we respectfully reclaim the word aliens as meaning beings from outer space. This question was from the original data reported in 2019.

$$\frac{5 \text{ aliens}}{7 \text{ power pouches}} = \frac{40 \text{ aliens}}{x \text{ power pouches}}$$

Maya



1=aliens
0=Power Pouches

Answer: 56

Aaliyah

$$5\overline{)7} \quad 1\tfrac{2}{5} \qquad 1.4 \times 40 = 56$$

$$\begin{array}{r} 5 \\ \hline 2 \end{array}$$

Answer: 56 power pouches

Aliens | 5 | 10 | 20 | 40 (×2, ×2, ×2)

Power Pouches | 7 | 14 | 28 | 56 (×2, ×2, ×2)

28 + 28 = 56

Answer: 56 Power pouches

Elijah

Liam

8 x 7 = 56

8 groups of 5 aliens so 8 groups x 7 power Pouches = 56

Answer: 56

$$\begin{array}{r} 7 \\ -\ 5 \\ \hline 2 \end{array} \qquad \begin{array}{r} 40 \\ +\ 2 \\ \hline 42 \end{array}$$

Answer: 42

Alex

| A | P |
|---|---|
| 5 | 7 |
| 10 | 14 |
| 15 | 21 |
| 20 | 28 (×2 =) |
| 40 | 56 |

$$\begin{array}{r} 14 \\ +\ 7 \\ \hline 21 \\ +\ 7 \\ \hline 28 \\ 28 \\ \hline 56 \end{array}$$

Answer: 56 power Pat

Jayden

$$\begin{array}{r} 7 \\ \times\ 5 \\ \hline 35 \\ +\ 40 \\ \hline 75 \end{array}$$

Answer: 75 power pouches.

Priya

| Aliens | Power Pouches |
|---|---|
| 5 | 7 |
| 10 | 14 |
| 15 | 21 |
| 20 | 28 |
| 25 | 35 |
| 30 | 42 |
| 35 | 49 |
| 40 | 56 |

For every 5 aliens it's 7 pp's so I just added 7 each time

Answer: 56 Power Pouches

Zoe

# Example 2:

## Comparison Proportion Problems ask the problem solver to determine whether or not two ratios are equivalent.

Version A-Given proportion is **incorrect**

A potion calls for 6 drops of magical Bulbadox Juice for every 9 cat hairs.

Neville squeezed the dropper too hard!

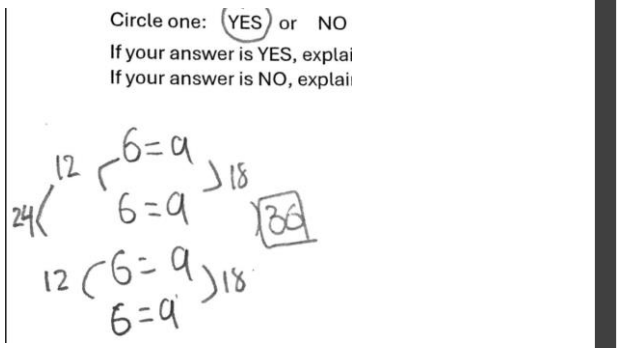
If there are 30 drops of magical Bulbadox Juice in his pot, should Neville use 36 cat hairs to make the potion have the same strength?

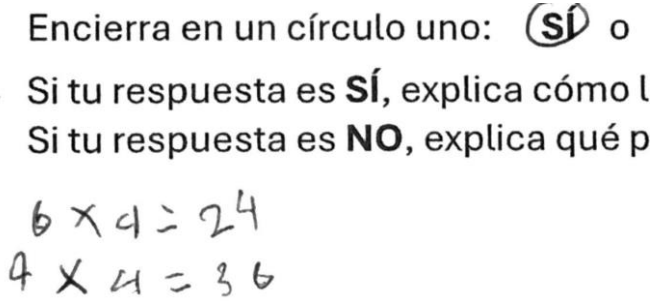
Version B-Given proportion is **correct**

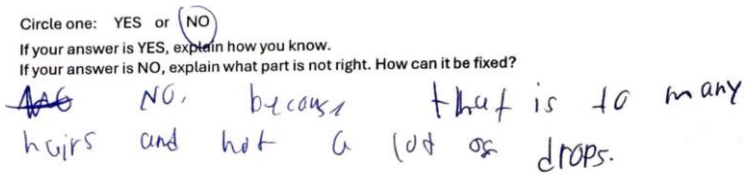
A potion calls for 6 drops of magical Bulbadox Juice for every 9 cat hairs.

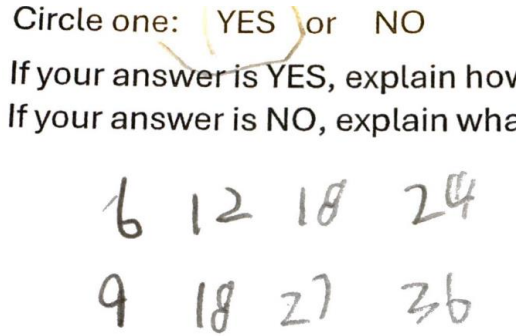
Neville squeezed the dropper too hard!

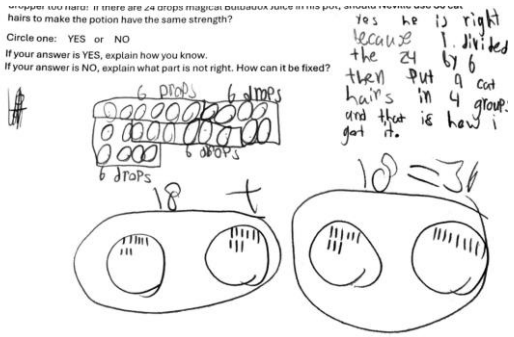
If there are 24 drops of magical Bulbadox Juice in his pot, should Neville use 36 cat hairs to make the potion have the same strength?




$$\frac{6 \text{ drops}}{9 \text{ cat hairs}} = \frac{24 \text{ drops}}{36 \text{ cat hairs}}$$

Equivalent Ratios

# The Plan--- Continued

During the requested leave, I will work closely with a co-author to finalize the manuscript, Instead of creating videos, I collaborated with two schools in two different states to collect additional data from 5-8$^{th}$ graders. This data expands the material of the book to include proportional reasoning comparison problem types.

# Accomplishments

- Obtained letter of interest  from Routledge
- Five writing retreats with co-authors
- Completed four book chapters- outlined the next four
- Expanded the original study to include the Comparison problems
- Completed data collection (1200 student surveys, 5th - 8th graders)
- Presented at two conferences on the book topic.
- Revised the book content plan, extending the material to include the new data
- Wrote and received a small university grant to engage student researchers (4 graduate and 2 undergraduate) in coding the new data set

# What I Learned

Teachers welcomed the opportunity to consider student thinking, problem solving, and applying this in the context of teaching proportion.

Translating research work and professional development into book format is challenging and takes time.

So far—Adding the comparison problem, comparing student strategies from the original missing value research results to strategies used in comparison problems is revealing and will be a useful addition for the book content.

Extending the project to include student researchers is a valuable addition to my original proposal and has multiple benefits beyond the original plan.

Thank you to my co-author, Olly Steinthorsdottitir.

# Next Steps: September 2026 to August 2027

- Continue to work with student researchers and co-author to complete the analysis of comparison problem surveys.
- Present with student researchers at conferences and offer the opportunity to participate in writing articles for publication.
- Finalize key components to complete the submission materials for the book to gain a contract agreement.
- While in negotiation with publishers, we may revisit the including of video as part of the assets in the book.