CASA — Texas A&M University Corpus Christi, Center for Academic Student Achievement

CALCULUS

Limits

Common Derivatives

$$\lim_{x\to a} f(x) = L \quad \text{if}$$
$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$$

$$\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x) \quad \text{if}$$
$$\lim_{x\to a} f(x) \text{ Does Not Exist}$$

L'Hospital's Rule

If $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\pm\infty}{\pm\infty}$ then,

$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$ a is a number, ∞ or $-\infty$

Derivatives

Definition and Notation

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Basic Properties and Formulas

$(fg)' = f'g + fg'$ – **Product Rule**

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ – **Quotient Rule**

$\frac{d}{dx}\left(x^n\right) = n x^{n-1}$ – **Power Rule**

$$\frac{d}{dx}\left(f\left(g(x)\right)\right) = f'\left(g(x)\right) g'(x)$$

This is the **Chain Rule**

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\cos^{-1} x\right) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1+x^2}$

$\frac{d}{dx}\left(a^x\right) = a^x \ln(a)$

$\frac{d}{dx}\left(\mathrm{e}^x\right) = \mathrm{e}^x$

$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}, \quad x > 0$

$\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x}, \quad x \neq 0$

$\frac{d}{dx}\left(\log_a(x)\right) = \frac{1}{x \ln a}, \quad x > 0$

Increasing/Decreasing

Concave Up/Concave Down

Critical Points

$x = c$ is a critical point of $f(x)$ provided either

1. $f'(c) = 0$ or **2.** $f'(c)$ doesn't exist.

Increasing/Decreasing

1. If $f'(x) > 0$ for all x in an interval I then $f(x)$ is increasing on the interval I.
2. If $f'(x) < 0$ for all x in an interval I then $f(x)$ is decreasing on the interval I.
3. If $f'(x) = 0$ for all x in an interval I then $f(x)$ is constant on the interval I.

Concave Up/Concave Down

1. If $f''(x) > 0$ for all x in an interval I then $f(x)$ is concave up on the interval I.
2. If $f''(x) < 0$ for all x in an interval I then $f(x)$ is concave down on the interval I.

Inflection Points

$x = c$ is a inflection point of $f(x)$ if the concavity changes at $x = c$.

The information for this handout was compiled from the following sources:
Paul's Online Math Notes. (n.d.). Retrieved from http://tutorial.math.lamar.edu/cheat_table.aspx

Calculus Formulas

1st Derivative Test

If $x = c$ is a critical point of $f(x)$ then $x = c$ is

1. a rel. max. of $f(x)$ if $f'(x) > 0$ to the left of $x = c$ and $f'(x) < 0$ to the right of $x = c$.
2. a rel. min. of $f(x)$ if $f'(x) < 0$ to the left of $x = c$ and $f'(x) > 0$ to the right of $x = c$.
3. not a relative extrema of $f(x)$ if $f'(x)$ is the same sign on both sides of $x = c$.

2nd Derivative Test

If $x = c$ is a critical point of $f(x)$ such that $f'(c) = 0$ then $x = c$

1. is a relative maximum of $f(x)$ if $f''(c) < 0$.
2. is a relative minimum of $f(x)$ if $f''(c) > 0$.
3. may be a relative maximum, relative minimum, or neither if $f''(c) = 0$.

Fundamental Theorem of Calculus

Part I: $g'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Part II: $\int_a^b f(x)\,dx = F(b) - F(a)$

Common Integrals

$\int k\,dx = kx + c$

$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$

$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + c$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$

$\int \ln u\,du = u\ln(u) - u + c$

$\int e^u\,du = e^u + c$

$\int \cos u\,du = \sin u + c$

$\int \sin u\,du = -\cos u + c$

$\int \sec^2 u\,du = \tan u + c$

$\int \sec u \tan u\,du = \sec u + c$

$\int \csc u \cot u\,du = -\csc u + c$

$\int \csc^2 u\,du = -\cot u + c$

$\int \tan u\,du = \ln|\sec u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c$

$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$

$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right) + c$

u Substitution :

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

Integration by Parts :

$$\int u\,dv = uv - \int v\,du \text{ and } \int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Products and (some) Quotients of Trig Functions

For $\int \sin^n x \cos^m x\,dx$ we have the following :

1. ***n* odd.** Strip 1 sine out and convert rest to cosines using $\sin^2 x = 1 - \cos^2 x$, then use the substitution $u = \cos x$.
2. ***m* odd.** Strip 1 cosine out and convert res to sines using $\cos^2 x = 1 - \sin^2 x$, then use the substitution $u = \sin x$.
3. ***n* and *m* both odd.** Use either 1. or 2.
4. ***n* and *m* both even.** Use double angle and/or half angle formulas to reduce the integral into a form that can be integrated.

For $\int \tan^n x \sec^m x\,dx$ we have the following :

1. ***n* odd.** Strip 1 tangent and 1 secant out and convert the rest to secants using $\tan^2 x = \sec^2 x - 1$, then use the substitution $u = \sec x$.
2. ***m* even.** Strip 2 secants out and convert rest to tangents using $\sec^2 x = 1 + \tan^2 x$, then use the substitution $u = \tan x$.
3. ***n* odd and *m* even.** Use either 1. or 2.
4. ***n* even and *m* odd.** Each integral will be dealt with differently.

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Calculus Formulas

Trig Substitutions :

$\sqrt{a^2-b^2x^2} \Rightarrow x=\frac{a}{b}\sin\theta$

$\cos^2\theta = 1-\sin^2\theta$

$\sqrt{b^2x^2-a^2} \Rightarrow x=\frac{a}{b}\sec\theta$

$\tan^2\theta=\sec^2\theta-1$

$\sqrt{a^2+b^2x^2} \Rightarrow x=\frac{a}{b}\tan\theta$

$\sec^2\theta=1+\tan^2\theta$

Partial Fractions :

Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$

Factor in $Q(x)$	Term in P.F.D
$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$
$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Area Between Curves :

$y=f(x) \Rightarrow A=\int_a^b \left(\text{upper function}\right)-\left(\text{lower function}\right)dx$

$x=f(y) \Rightarrow A=\int_c^d \left(\text{right function}\right)-\left(\text{left function}\right)dy$

Volumes of Revolution :

$V=\int A(x)\,dx$ and $V=\int A(y)\,dy$

Rings

$A=\pi\left((\text{outer radius})^2-(\text{inner radius})^2\right)$

Cylinders

$A=2\pi(\text{radius})(\text{width / height})$

Work :

$W=\int_a^b F(x)\,dx$

Average Function Value :

$$f_{avg}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

Arc Length Surface Area :

$SA=\int_a^b 2\pi y\,ds$ (rotate about x-axis)

$SA=\int_a^b 2\pi x\,ds$ (rotate about y-axis)

Improper Integral

Infinite Limit

1. $\int_a^\infty f(x)\,dx=\lim_{t\to\infty}\int_a^t f(x)\,dx$
2. $\int_{-\infty}^b f(x)\,dx=\lim_{t\to-\infty}\int_t^b f(x)\,dx$
3. $\int_{-\infty}^\infty f(x)\,dx=\int_{-\infty}^c f(x)\,dx+\int_c^\infty f(x)\,dx$

Discontinuous Integrand

1. Discont. at a: $\int_a^b f(x)\,dx=\lim_{t\to a^+}\int_t^b f(x)\,dx$
2. Discont. at b: $\int_a^b f(x)\,dx=\lim_{t\to b^-}\int_a^t f(x)\,dx$
3. Discontinuity at $a<c<b$:

$\int_a^b f(x)\,dx=\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$

Comparison Test for Improper Integrals :

If $f(x)\ge g(x)\ge 0$ on $[a,\infty)$ then,

1. If $\int_a^\infty f(x)\,dx$ conv. then $\int_a^\infty g(x)\,dx$ conv.
2. If $\int_a^\infty g(x)\,dx$ divg. then $\int_a^\infty f(x)\,dx$ divg

Useful fact : If $a>0$ then

$\int_a^\infty \frac{1}{x^p}\,dx$ converges if $p>1$

and diverges for $p\le 1$

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